

(d) $\mathbf{U} + \mathbf{W}$

4. Andaikan \mathbf{U} dan \mathbf{W} di dalam \mathbf{R}^4 , di mana \mathbf{U} dan \mathbf{W} masing-masing dibangun oleh himpunan vektor-vektor :

$$\mathbf{U} = \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 3 \\ -1 \end{pmatrix} \right\} \text{ dan } \mathbf{W} = \left\{ \begin{pmatrix} 2 \\ 3 \\ 2 \\ -3 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 2 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 4 \\ -3 \end{pmatrix} \right\}$$

Carilah basis dan dimensi dari :

a) \mathbf{U}

b) \mathbf{W}

c) $\mathbf{U} + \mathbf{W}$

d) $\mathbf{U} \cap \mathbf{W}$

5. Suppose \mathbf{U} and \mathbf{W} are subspace of $\mathbf{V} = \mathbf{R}^4$.

$$\mathbf{U} = \left\{ \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \mid a = c + 2d, b + c = d; a, b, c, d \in \mathbf{R} \right\}$$

$$\mathbf{W} = \left\{ \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \mid a - b = c, c + 2b = d; a, b, c, d \in \mathbf{R} \right\}$$

find basis and dimension of :

(a) \mathbf{U}

(b) \mathbf{W}

(c) $\mathbf{U} \cap \mathbf{W}$

(d) $\mathbf{U} + \mathbf{W}$

6. Suppose \mathbf{U} and \mathbf{W} are subspace of \mathbf{V} in \mathbf{R}^5 .

$$\mathbf{U} = \left\{ \begin{pmatrix} a \\ b \\ c \\ d \\ e \end{pmatrix} \mid a - 2b + c = 2d, a - b = d + 2e; a, b, c, d, e \in \mathbf{R} \right\}$$

$$\mathbf{W} = \left\{ \begin{pmatrix} a \\ b \\ c \\ d \\ e \end{pmatrix} \mid b = c - 2d + e, b - c = 2e; a, b, c, d, e \in \mathbf{R} \right\}$$

Find basis and dimension of :

a) $U \cap W$

b) $U + W$

7. Andaikan U dan W di dalam \mathbf{R}^4 , di mana U dan W masing-masing dibangun oleh himpunan vektor-vektor :

$$U = \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 3 \\ -1 \end{pmatrix} \right\} \text{ dan } W = \left\{ \begin{pmatrix} 1 \\ 2 \\ 2 \\ -2 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 2 \\ -3 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 4 \\ -3 \end{pmatrix} \right\}$$

Carilah basis dan dimensi dari :

(a) $U + W$

(b) $U \cap W$

8. Andaikan $V = \mathbf{R}^5$ serta U dan W adalah *subspace* dari V dengan

$$U = \left\{ \begin{pmatrix} a \\ b \\ c \\ d \\ e \end{pmatrix} \mid a - 2e = d, b + e = c; a, b, c, d, e \in \mathbf{R} \right\}.$$

$$W = \left\{ \begin{pmatrix} a \\ b \\ c \\ d \\ e \end{pmatrix} \mid b = 2d - e, a + e = 2c; a, b, c, d, e \in \mathbf{R} \right\}.$$

Carilah basis dan dimensi dari :

a) U

b) W

c) $U \cap W$

d) $U + W$

9. Suppose U and W are subspace of V in \mathbf{R}^5 .

$$U = \left\{ \begin{pmatrix} a \\ b \\ c \\ d \\ e \end{pmatrix} \mid a - 2b + c = 2d, a - b = d + 2e; a, b, c, d, e \in \mathbf{R} \right\}$$

$$W = \left\{ \begin{pmatrix} a \\ b \\ c \\ d \\ e \end{pmatrix} \mid b = c - 2d + e, b - c = 2e; a, b, c, d, e \in \mathbb{R} \right\}$$

Find basis and dimension of :

a) $U \cap W$

b) $U + W$

10. Suppose U and W are subspaces of \mathbb{R}^5 ; where $U = \left\{ \begin{pmatrix} a \\ b \\ c \\ d \\ e \end{pmatrix} \mid b + c - 2d = e; \text{ with } a, b, c, d, e \in \mathbb{R} \right\}$

R } and $W = \left\{ \begin{pmatrix} a \\ b \\ c \\ d \\ e \end{pmatrix} \mid a - 3b + 2e = 0, c = d - 2e; \text{ with } a, b, c, d, e \in \mathbb{R} \right\}$. Find basis and dimension

of (i) U , (ii) W , (iii) $U \cap W$, dan (iv) $U + W$

11. Andaikan U dan W adalah subspaces dari \mathbb{R}^5 ; di mana $U = \left\{ \begin{pmatrix} a \\ b \\ c \\ d \\ e \end{pmatrix} \mid b + c + d + e = 0 \right\}$ dan $W =$

$\left\{ \begin{pmatrix} a \\ b \\ c \\ d \\ e \end{pmatrix} \mid a + b = 2e, a + c = d - 2e \right\}$. Carilah basis dan dimensi dari ruang (i) U , (ii) W , (iii) $U \cap W$,

dan (iv) $U + W$

12. Andaikan U dan W merupakan *subspace* dari \mathbb{R}^5 ; di mana :

$$\mathbf{U} = \{u_1, u_2, u_3\}, \text{ dengan } u_1 = \begin{pmatrix} 1 \\ 3 \\ -2 \\ 2 \\ 3 \end{pmatrix}, u_2 = \begin{pmatrix} 1 \\ 4 \\ -3 \\ 4 \\ 2 \end{pmatrix}, \text{ dan } u_3 = \begin{pmatrix} 1 \\ 3 \\ 0 \\ 2 \\ 3 \end{pmatrix}.$$

$$\mathbf{W} = \{w_1, w_2, w_3\}, \text{ di mana } w_1 = \begin{pmatrix} 2 \\ 3 \\ -1 \\ -2 \\ 9 \end{pmatrix}, w_2 = \begin{pmatrix} 1 \\ 5 \\ -6 \\ 6 \\ 1 \end{pmatrix}, \text{ dan } w_3 = \begin{pmatrix} 2 \\ 4 \\ 4 \\ 2 \\ 8 \end{pmatrix}.$$

Carilah basis dan dimensi dari :

(a). $\mathbf{U} + \mathbf{W}$

(b). $\mathbf{U} \cap \mathbf{W}$

13. Let $\mathbf{U} = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \mid x + 2y - z = 0; x, y, z \in \mathbb{R} \right\}$ and $\mathbf{W} = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \mid 2x + 3y + z = 0; x, y, z \in \mathbb{R} \right\}$.

Find basis and dimension of :

a. \mathbf{U} b. \mathbf{W} c. $\mathbf{U} \cap \mathbf{W}$

14. Andaikan ruang vektor $\mathbf{U} = \left\{ \begin{pmatrix} 1 \\ 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 3 \\ 4 \end{pmatrix} \right\}$, dan $\mathbf{W} = \left\{ \begin{pmatrix} 1 \\ -1 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 4 \\ -1 \\ 3 \\ 0 \end{pmatrix} \right\}$.

Carilah basis dan dimensi dari :

(a). \mathbf{U} , (b) \mathbf{W} , (c) $\mathbf{U} + \mathbf{W}$, (d). $\mathbf{U} \cap \mathbf{W}$.

15. Prove : If \mathbf{U} and \mathbf{W} are subspace of \mathbf{V} , then $\mathbf{U} + \mathbf{W}$ are subspace of \mathbf{V} .